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# Combined forced–free laminar heat transfer to a highly heated gas in a vertical annulus

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**Abstract**—Numerical investigation of the combined forced–free laminar convection (both upward and downward flow), with a simultaneously developing hydrodynamic and thermal boundary layer in an annulus, is considered. The thermal condition of an inner wall is isothermal or constant heat flux; that of an outer wall is adiabatic. Fluid properties are varied as a function of temperature. Heat transfer characteristics are obtained by solving the continuity, momentum, energy and integral continuity equations on the basis of the boundary-layer approximation. Particular attention is given to the critical condition for flow reversal and the effect of property variations on both Nusselt number and friction factor.

## 1. INTRODUCTION

IN A High Temperature Gas Cooled Reactor (HTGR), the reactor core is designed to achieve a high outlet temperature. Helium gas coolant flows through annular channels between fuel rods and inner walls of holes in hexagonal graphite blocks. Fuel element wall temperature is very high because of the relatively inferior heat transfer characteristics of a gas. It is therefore necessary to investigate the effect of large temperature differences between wall and fluid on heat transfer and hydrodynamics for an annular passage. In normal conditions, helium gas flows downward in the reactor core. In abnormal conditions such as emergency cooling, the direction of flow might be reversed owing to buoyant force. In view of HTGR core safety it is, therefore, of practical importance to investigate the combined forced–free convection heat transfer in a vertical annulus.

Much research has been done to investigate theoretically and experimentally the effect of free convection on laminar forced–free convection in various passages under various conditions. Scheele and Rosen [1] presented numerical solutions for the effect of free convection on fully-developed laminar flow in a vertical tube. Greene and Scheele [2] calculated the combined flow by considering the temperature dependence of viscosity. Lawrence and Chato [3] obtained numerical results for the developing combined laminar flow in a vertical tube with a uniform entering velocity profile and wall heat flux. Zeldin and Schmidt [4] made numerical analysis for the developing combined laminar flow in an isothermal vertical tube. Siegwarth and Hanratty [5] studied the flow in a horizontal tube experimentally and analytically. Quintiere and Mueller [6] examined the combined laminar flow between vertical parallel plates. Only a few investigations have been published on the combined forced–free laminar flow in an annular channel. Maitra and Raju [7] and El-Shaarawi and Sarhan [8] presented the solutions for the combined numerical results for the developing combined flow in a vertical annulus, where one of the walls was assumed to be adiabatic, and the other isothermal.

Investigations with temperature dependence of fluid properties were made for the developing forced laminar flow in a parallel passage by Swearingen and McEligot [9], and in an annulus by Shumway and McEligot [10].

To the authors' knowledge, however, no data are available for the developing combined forced–free laminar heat transfer with temperature dependence of fluid properties.

In the present study, numerical investigations of the combined forced–free convection (both upward and downward flow), with a simultaneously developing hydrodynamic and thermal boundary layer in an annulus, are considered. The thermal condition of an inner wall is isothermal or constant heat flux; while that of an outer wall is adiabatic. The property variation of a coolant gas is taken into account.

## 2. GOVERNING EQUATION AND NUMERICAL METHOD OF SOLUTION

The present analysis assumes steady, laminar, axially symmetric and incompressible Newtonian fluid. The axial conduction of heat is neglected. Governing differential equations are developed on the basis of the standard boundary-layer approximation. Fluid properties are varied as a function of temperature. The conservation equations of mass, momentum and energy in the entry region of a vertical annulus become:

$$\frac{\partial}{\partial z}(\rho u)+\frac{1}{r}\frac{\partial}{\partial r}(\rho r v) = 0 \qquad (1)$$

$$\rho\left(u\frac{\partial u}{\partial z}+v\frac{\partial u}{\partial r}\right) = -\frac{\partial P}{\partial z}+\frac{1}{r}\frac{\partial}{\partial r}\left(\mu r\frac{\partial u}{\partial r}\right)\pm\rho g \qquad (2)$$

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## NOMENCLATURE

- $c_p$ specific heat
- $De$ hydraulic diameter of annulus, $2(r_2 - r_1)$
- $f$ friction factor, $(De/4\Delta z)(\Delta P - \Delta P_{acc} \pm \rho g \Delta z)/\frac{1}{2}\rho_m u_m^2$
- $g$ gravitational body force per unit mass
- $Gr$ $Gr_t$ or $Gr_q$
- $Gr_q$ Grashof number (constant heat flux condition), $De^4 g q \beta/\nu^2 \lambda$
- $Gr_t$ Grashof number (isothermal wall condition), $De^3 g \Delta\rho/\rho\nu^2$
- $h$ heat transfer coefficient
- $N$ annulus radius ratio, $r_1/r_2$
- $Nu$ Nusselt number, $hDe/\lambda$
- $P$ pressure
- $P'$ pressure defect, $P - P_i \mp \rho_i g z$
- $\Delta P$ pressure drop in axial direction
- $\Delta P_{acc}$ acceleration defect, $\int_{r_1}^{r_2} \rho u^2 r \, dr \Big/ \int_{r_1}^{r_2} r \, dr$
- $Pr$ Prandtl number, $\mu c_p/\lambda$
- $q$ heat flux at wall
- $r$ radial coordinate
- $R$ dimensionless radial coordinate, $r/r_2$
- $Re$ Reynolds number, $\rho u De/\mu$
- $r_1$ inner radius of annulus
- $r_2$ outer radius of annulus
- $t$ temperature
- $t_m$ mixed mean temperature, $\int_{r_1}^{r_2} utr \, dr \Big/ \int_{r_1}^{r_2} ur \, dr$
- $t_i$ entrance temperature
- $t_w$ wall temperature
- $u$ axial velocity component
- $U$ dimensionless axial velocity, $u/u_0$
- $u_m$ mean axial velocity, $\int_{r_1}^{r_2} ur \, dr \Big/ \int_{r_1}^{r_2} r \, dr$
- $u_0$ entrance velocity
- $v$ radial velocity component
- $z$ axial coordinate
- $Z$ dimensionless axial coordinate, $2z(1-N)/r_2 Re_i$
- $Zr$ critical distance for zero velocity gradient normal to a wall.

Greek symbols

- $\beta$ thermal expansion coefficient, $t^{-1}$
- $\lambda$ thermal conductivity
- $\mu$ viscosity
- $\nu$ kinematic viscosity
- $\rho$ density.

Subscripts

- m physical property evaluated at $t_m$
- q constant heat flux
- t isothermal wall
- w heated (inner) wall condition
- i inlet condition
- 1 inner wall
- 2 outer wall.

Superscript

- * dimensionless quantity.

$$0 = -\frac{\partial P}{\partial r} \tag{3}$$

$$c_p \rho \left( u \frac{\partial t}{\partial z} + v \frac{\partial t}{\partial r} \right) = \frac{1}{r} \frac{\partial}{\partial r} \left( \lambda r \frac{\partial t}{\partial r} \right). \tag{4}$$

The plus and minus signs in the buoyancy term of equation (2) refer to downward and upward flow, respectively. An additional equation necessary for the numerical solution is the integral continuity equation:

$$2\pi \int_{r_1}^{r_2} \rho u r \, dr = \pi (r_2^2 - r_1^2). \tag{5}$$

These equations are solved using the inlet condition of uniform entering velocity and temperature profiles:

$$\text{at} \quad z = 0; \; u = u_0, \quad v = 0, \quad t = t_i, \quad P = P_i. \tag{6}$$

Boundary conditions are as follows:

$$\begin{aligned} &\text{at} \quad r = r_1 \text{ or } r_2; && u = v = 0 && \text{(no slip)} \\ &r = r_1; && t = t_w && \text{(isothermal)} \\ &\text{or} && -\lambda\, \partial t/\partial r = q_w && \text{(constant heat flux)} \\ &r = r_2; && \partial t/\partial r = 0 && \text{(adiabatic).} \end{aligned} \tag{7}$$

The radius ratio $N$ $(= r_1/r_2)$ is fixed at 0.9 in the present calculations.

Equations (1)–(5) have been numerically solved by means of an implicit finite-difference scheme. To accommodate the nonlinearities of these equations it is necessary to iterate the solution sequence. This iterative scheme was discussed by Swearingen and McEligot [9]. If the temperature dependence of fluid properties (except that of the density in the gravitational body force term) is assumed to be negligible, $Gr/Re$ is the only parameter to express the effect of free convection according to the Boussinesq approximation. In the present study also, $Gr/Re$ is the primary parameter for the effect of free convection. However, in the case of a high heating rate, temperature dependence of the other properties is not negligible, and thus the solution cannot be expressed by $Gr/Re$ alone. The transport properties of gas, i.e. the viscosity and the thermal conductivity, are nearly independent of pressure and can be approximated as being proportional to $t^n$, where $t$ is the absolute temperature and $n$ is an empirical constant. Density of gas is inversely proportional to the pressure; but, as the axial pressure drop in a tube is usually much less than the absolute pressure, the density also can be approximated as a function of $t$ only. With these approximations, the effect of the temperature dependence of fluid properties can be expressed by an additional parameter $t_w/t_i$ for the isothermal condition or $qDe/\lambda_i t_i$ for the constant wall heat flux. Nondimensional forms of the equations are presented in the Appendix. Heat transfer and fluid flow can thus be expressed by $(Gr/Re)_i$ together with $t_w/t_i$ or $qDe/\lambda_i t_i$.

The exponent $n$ is about 0.67 for helium and about 0.7 for air; slightly different values are found for other kinds of gases. The Prandtl number of helium is 0.671 and almost independent of temperature and other gases also have a roughly constant value of $Pr$ of around 0.7. The present calculations are made for helium; however, results obtained will be roughly applicable for gases other than helium.

The local Nusselt and Reynolds numbers are defined as

$$Nu = hDe/\lambda_m, \quad Re = \rho_m Deu_m/\mu_m$$

where the subscript m indicates that the fluid properties are evaluated at the mixed mean temperature.

The friction factor is defined as

$$f = \frac{De}{4\Delta z}(\Delta P - \Delta P_{acc} \pm \rho_m g \Delta z)/\tfrac{1}{2}\rho_m u_m^2 \quad (8)$$

$\Delta P_{acc}$ is the term due to acceleration, given by

$$\Delta P_{acc} = \Delta \int_{r_2}^{r_1} \rho u^2 r \, dr \Big/ \int_{r_2}^{r_1} r \, dr. \quad (9)$$

In the calculations, a sufficiently large number of mesh points are distributed in both radial and axial directions, so that the mesh size has practically no influence on the results. Convergence criteria for velocity and temperature are $10^{-5}$.

## 3. RESULTS AND DISCUSSIONS

### 3.1. *Velocity profiles*

Figure 1 represents an example of $(Gr_t/Re)_i$ dependence of axial velocity profile $U$ for both upward and downward flows at $Z = 10^{-4}$. This figure shows that when the free convection opposes the forced flow (i.e. downward flow) the buoyancy force retards the

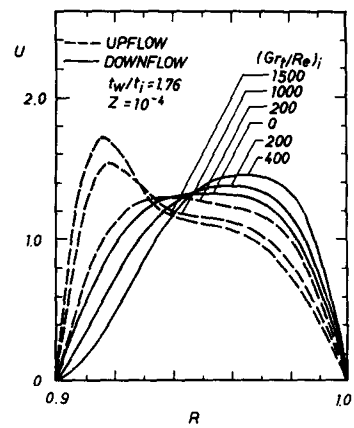


FIG. 1. $(Gr_t/Re)_i$ dependence of axial velocity profiles.

fluid near the heated boundary (i.e. inner wall). On the other hand, when the free convection aids the forced flow (i.e. upward flow) the fluid accelerates near the heated wall and decelerates near the opposite adiabatic boundary. In the present analysis, the same velocity profile characteristics as presented by El-Shaarawi and Sarhan [8] are obtained.

In the case of the downward-flow problem, a maximum distortion of velocity profile occurs when a gradient of the profile at the heat transfer boundary reaches its minimum value. Therefore, a flow reversal takes place near the heated wall. Figure 2 represents the $(Gr_t/Re)_i$ dependence of its velocity gradient normal to the wall at the heated boundary corresponding to the same value of the additional parameter $(t_w/t_i)$. With increasing $(Gr_t/Re)_i$, the gradient of a developing axial velocity at the heated wall deviates from the profile for $(Gr_t/Re)_i = 0$ until a location where its value represents a minimum. With increasing axial distance further, the gradient recovers and approaches the fully-developed isothermal profile as the fluid temperature approaches the isothermal wall temperature. Flow reversal takes place near a location where the gradient of axial velocity profile vanishes or becomes negative. Under such a condition, boundary-layer separation may

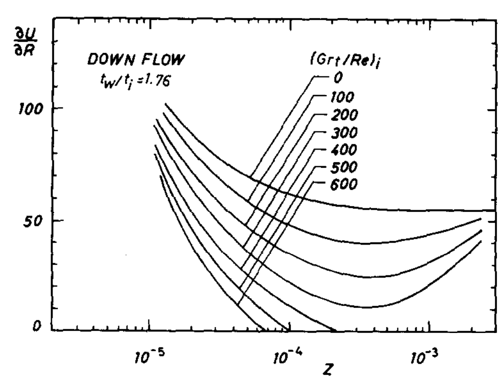


FIG. 2. $(Gr_t/Re)_i$ dependence of axial velocity gradient normal to the wall, downflow.

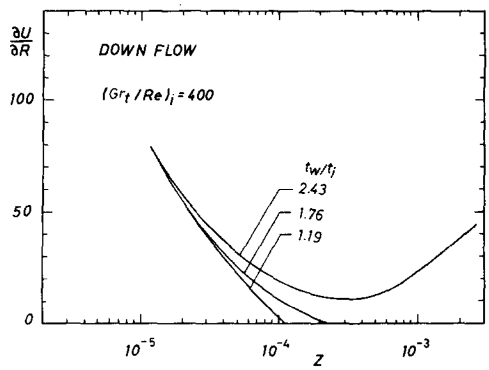


FIG. 3. $t_w/t_i$ dependence of axial velocity gradient normal to the wall, downflow.

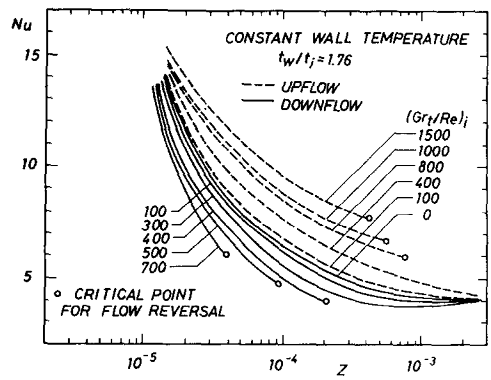


FIG. 5. $(Gr_t/Re)_i$ dependence of Nusselt number.

occur and the boundary-layer approximation might no longer be applicable. Heat transfer and fluid flow beyond the flow reversal is thus out of the present scope and is left for future work.

In case of the upward flow, flow reversal takes place on the adiabatic wall in contrast with the downward flow.

The effect of fluid property variations, i.e. effect of $(t_w/t_i)$, is shown in Fig. 3. For a higher $(t_w/t_i)$, the velocity gradient does not fall to zero. This is due to an increasing thermal conductivity of gas for higher $(t_w/t_i)$. Thus, the condition of the flow reversal is influenced appreciably if the property variations of gas are considered. In case of constant wall heat flux, the same tendency as constant wall temperature has been obtained.

### 3.2. *Critical condition for flow reversal*

El-Shaarawi and Sarhan [8] presented the critical distance $Zr$, where a velocity gradient normal to the wall vanishes. In the present study, $Zr$ is obtained for each value of $(Gr/Re)_i$, $t_w/t_i$ or $qDe/\lambda_i t_i$. For a small $t_w/t_i$, i.e. for negligible property variations, the present values of $Zr$ are almost in agreement with those of El-Shaarawi and Sarhan. When the property variations are appreciable, the point of zero velocity gradient moves downwards and disappears with increase in $t_w/t_i$ or $qDe/\lambda_i t_i$. Figure 4 represents the critical condition for the flow reversal. The flow reversal disappears with decreasing $(Gr/Re)_i$ or with increasing $t_w/t_i$, $qDe/\lambda_i t_i$. It is noticeable that the critical $(Gr/Re)_i$ of downward flow is much smaller than that of upward flow. The following consideration is responsible for this tendency; that is, in case of downward flow, buoyancy force directly retards the fluid near the heated boundary. On the other hand, in case of upward flow, the force retards the fluid near the adiabatic boundary only indirectly through the acceleration near the heated boundary.

The critical conditions for flow reversal can be represented by following relations:

Uniform isothermal condition ($1 < t_w/t_i < 2.5$)

$$(Gr/Re)_i = 420 + 200(t_w/t_i) \quad \text{(upward flow)}$$
$$(Gr/Re)_i = 40 + 190(t_w/t_i) \quad \text{(downward flow).} \tag{10}$$

Uniform wall heat flux condition ($0 < qDe/\lambda_i t_i < 6.0$)

$$(Gr/Re)_i = 3750 + 1050(qDe/\lambda_i t_i) \quad \text{(upward flow)}$$
$$(Gr/Re)_i = 640 + 610(qDe/\lambda_i t_i) \quad \text{(downward flow).} \tag{11}$$

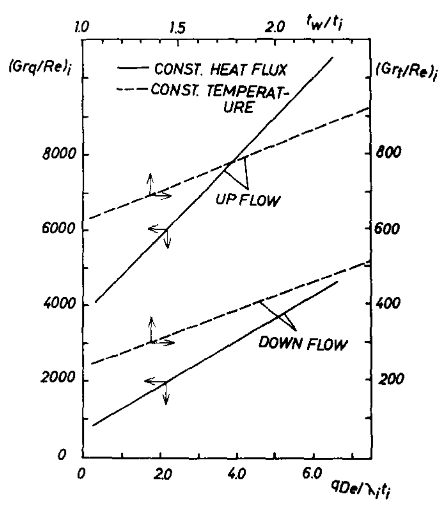


FIG. 4. Critical conditions for flow reversal.

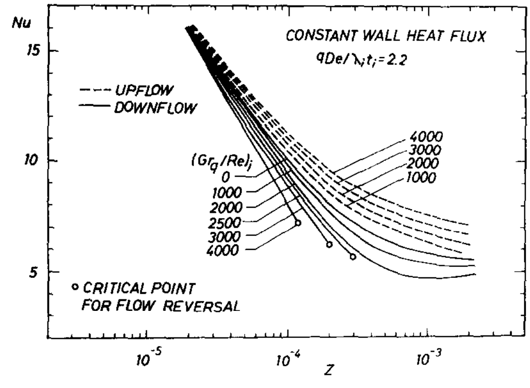


FIG. 6. $(Gr_q/Re)_i$ dependence of Nusselt number.

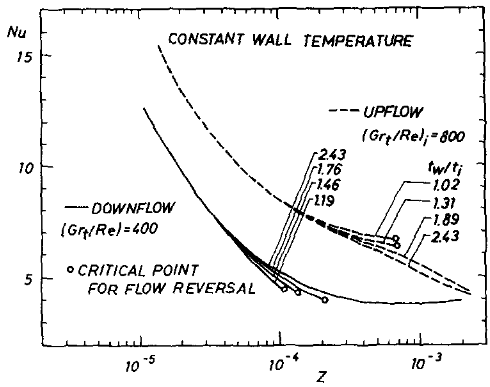


FIG. 7. $t_w/t_i$ dependence of Nusselt number.

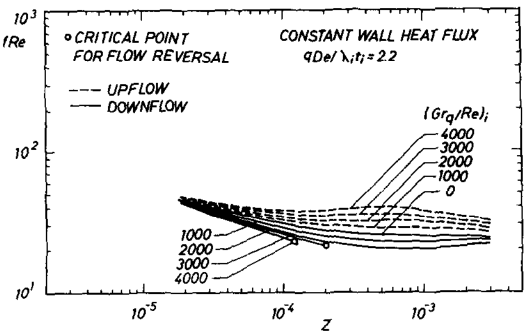


FIG. 9. $(Gr_q/Re)_i$ dependence of friction factor.

### 3.3. *Heat transfer*

The effect of free convection on laminar heat transfer is shown in Figs. 5 and 6, where the thermal conditions of an inner wall are isothermal and uniform heat flux, respectively. These figures show that with increasing $(Gr/Re)_i$ Nusselt number decreases for downward flow and increases for upward flow. With increasing the axial distance further, the Nusselt number approaches a fully-developed isothermal or uniform heat flux value. This tendency is the same as the one obtained by El-Shaarawi and Sarhan. Figure 7 shows the effect of property variations on a combined convection heat transfer for various $t_w/t_i$ in the case of isothermal wall condition. It is seen that the Nusselt number is almost independent of $t_w/t_i$. This tendency for property variations is very similar to that of laminar forced convection heat transfer [9, 10]. In case of the uniform heat flux heating, the same tendency as that of the isothermal wall heating has been obtained.

### 3.4. *Friction factor*

The effect of free convection on laminar friction factor is given in Figs. 8 and 9 with $(Gr_t/Re_i)$ and $(Gr_q/Re)_i$ as parameters, respectively. These figures show that the product of the friction factor and the Reynolds number increases with increasing $(Gr/Re)_i$ except for the down flow of uniform heat flux heating. With increasing axial distance, the product approaches a fully-developed value.

Figure 10 presents the effect of the property variations with the additional parameter $t_w/t_i$ in the case of isothermal wall heating. The effect of the property variations is small for the downward flow; while it is relatively large for the upward flow. The same tendency has been obtained for the other values of the parameter.

Calculation of the total pressure drop is often required in engineering applications. The total pressure drop in the direction of flow can be obtained from equation (8), using the friction factor, $f$, as

$$\Delta P = (4f)\tfrac{1}{2}\rho_m u_m^2\left(\frac{\Delta z}{De}\right) + \Delta P_{acc} \pm \rho_m g \Delta z, \quad (12)$$

where the plus and minus signs refer to upward and downward flows, respectively. In engineering calculations, the acceleration defect $\Delta P_{acc}$ is usually approximated as

$$\Delta P_{acc} \approx \Delta(\rho_m u_m^2) = (\rho u)_i^2 \Delta(\rho_m^{-1}). \quad (13)$$

## 4. CONCLUSIONS

Numerical investigations of the laminar combined forced–free convection heat transfer were studied. The properties of the gas were varied as a function of temperature. Results obtained are summarized below:

(1) Combined forced–free laminar Nusselt number, friction factor and critical conditions for flow reversal in case of both isothermal and uniform heat flux wall conditions were obtained.

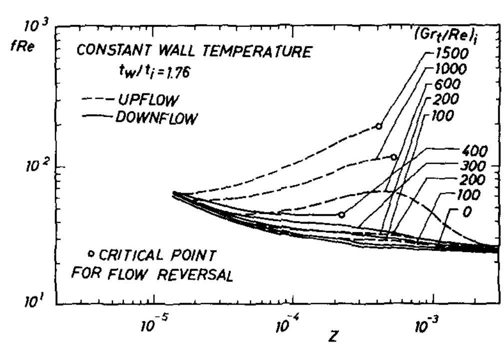


FIG. 8. $(Gr_t/Re)_i$ dependence of friction factor.

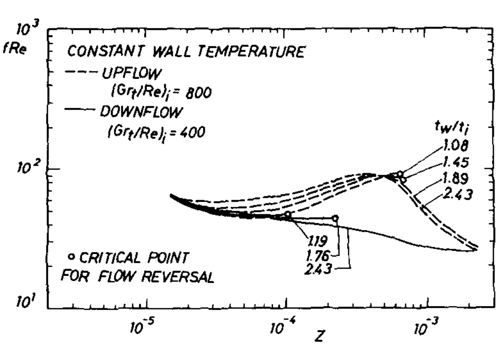


FIG. 10. $t_w/t_i$ dependence of friction factor.

(2) The fluid-property variation influences rather slightly the heat transfer coefficient or friction factor. The influence is, however, significant for the critical condition of the flow reversal.
(3) With increase in $(Gr/Re)_i$, Nusselt number decreases for downward flow and increases for upward flow. The product of friction factor and Reynolds number also increases except for the downward flow of constant wall heat flux.
(4) The flow reversal takes place when $(Gr/Re)_i$ exceeds a critical value. For a given $(Gr/Re)_i$, however, the flow reversal disappears with increase in $t_w/t_i$ or $qDe/\lambda_i t_i$.

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## APPENDIX

Non-dimensional forms of the fundamental equations are given below. Nondimensional variables are defined as

$$Z = z/z_0, \quad z_0 = r_2 Re_i/2(1-N)$$

$$R = r/r_2$$

$$u^* = u/u_0$$

$$v^* = v/v_0, \quad v_0 = \rho_i r_2/2(1-N)\mu_i$$

$$t^* = \begin{cases} (t-t_i)/(t_w-t_i) & \text{(isothermal)} \\ (t-t_i)/(qDe/\lambda_i) & \text{(constant heat flux)} \end{cases}$$

$$P^* = P'/P_0, \quad P_0 = \rho_i u_0^2.$$

Variation of physical properties is approximated as

$$\rho^* = \rho_i \theta^{-1}$$

$$\mu^* = \mu_i \theta^{n_1}$$

$$\lambda^* = \lambda_i \theta^{n_2}$$

$$c_p^* = c_{pi} \theta^{n_3},$$

where $\theta$ is a supplemental nondimensional temperature to represent property variation.

$$\theta = t/t_i$$

$$= \begin{cases} (t_w/t_i - 1)t^* + 1 & \text{(isothermal wall)} \\ (qDe/\lambda_i)t^* + 1 & \text{(constant heat flux).} \end{cases}$$

As $t_w/t_i \to 1$ or $q \to 0$, $\theta$ tends to unity; thus property variation becomes negligible.

Using these variables, the fundamental equations can be nondimensionalized as

$$\frac{1}{R}\frac{\partial}{\partial R}(\rho^* R v^*) + \frac{\partial}{\partial Z}(\rho^* u^*) = 0$$

$$\rho^*\left(u^*\frac{\partial u^*}{\partial Z} + v^*\frac{\partial u^*}{\partial R}\right) = -\frac{\partial P^*}{\partial Z} + \frac{1}{R}\frac{\partial}{\partial R}\left(\mu^* R\frac{\partial u^*}{\partial R}\right)$$

$$\pm \begin{cases} \left(\dfrac{Gr_t}{Re}\right)_i \dfrac{1}{4(1-N)^2}\left(\dfrac{t_w}{t_i}\right)\dfrac{t^*}{\theta} & \text{(isothermal wall)} \\ \left(\dfrac{Gr_q}{Re}\right)_i \dfrac{1}{4(1-N)^2}\dfrac{t^*}{\theta} & \text{(constant heat flux)} \end{cases}$$

$$c_p^*\rho^*\left(u^*\frac{\partial t^*}{\partial Z} + v^*\frac{\partial t^*}{\partial R}\right) = \frac{1}{Pr_i}\frac{1}{R}\frac{\partial}{\partial R}\left(\lambda^* R\frac{\partial t^*}{\partial R}\right).$$

Boundary conditions are

at $Z = 0$; $u^* = 1$, $v^* = 0$, $t^* = 0$, $P^* = 0$

at $R = N$ and $1$; $u^* = v^* = 0$

$R = N$; $t^* = 1$ (isothermal wall)

or $-\lambda^*\dfrac{\partial t^*}{\partial R} = \dfrac{1}{2(1-N)}$ (constant heat flux)

$R = 1$; $\dfrac{\partial t^*}{\partial R} = 0$

Thus, the solutions of $u^*$, $v^*$, $t^*$ can be expressed as a function of following co-ordinates and parameters;

co-ordinates: $Z, R$

fundamental non-dimensional numbers:

$(Gr_t/Re)_i$ or $(Gr_q/Re)_i$, $Pr_i$

geometrical parameter: $N$

property parameters: $n_1, n_2, n_3$

heating parameters: $t_w/t_i$ or $qDe/\lambda_i t_i$.

## TRANSFER DE CHALEUR D'ECOULEMENT LAMINAIRE DE LA CONVECTION MIXTE FORCEE/LIBRE AU GAZ FORTEMENT CHAUFFE DANS LE CONDUIT VERTICAL ANNULAIRE

**Résumé**—On analyse les valeurs numériques de l'écoulement laminaire de la convection mixte forcée/libre (des courants ascendant et descendant) sur le cas où se développent simultanément le courant dans le conduit annulaire et la couche limite des températures. La condition thermique du mur intérieur est supposée comme le mur isotherme ou le flux isotherme et celle du mur extérieur comme adiabatique. On fait varier des valeurs physiques du fluid en fonction des températures. Basé sur l'approximation d'écoulement laminaire, on obtient les caractéristiques de transfer de chaleur par résolution des équation de continuité, celle de mouvement, celle d'énergie et celle de continuité intégrée. On examine surtout des conditions génératrices du courant inverse et l'influence de la variation des valeurs physiques à l'égard des nombre de Nusselt et coefficient de frottement.

## WÄRMEÜBERTRAGUNG BEI LAMINARER STRÖMUNG UNTER ERZWUNGENER UND FREIER MISCHSTRÖMUNG AUF STARK ERHITZT GAS IM SENKRECHTEN RINKSPALT

**Zusammenfassung**—Die erzwunger und freier mischströmung im senkrechten Rinkspalt wird numerisch unter laminaren Strömungsbedingungen für den Fall einer gleichzeiten Entwicklung von Strömung und Temperaturgrenzschicht analysiert. Die Wärmebedingung an der Innenwandung ist isothermisch oder gleichmässiger Wärmeströmdicht. Stoffwerte für das Gas wurden in funktionäler Abhängigkeit von Temperature verändert. Die Wärmeübertragung wird durch Annäherung an die Grenzschicht durch Lösung des Gleichungssystems für Kontinuitäts, Impuls und Energiesatz bestimmt. Insbesonders wurde der Einfluss von Veränderung der physikalischen Grössen auf die Bedingungen zur Gegenstrombildung auf die Nusseltzahl und auf den Reibungs koeffizienten untersucht.

## ТЕПЛООТДАЧА К ЛАМИНАРНОМУ ПОТОКУ СИЛЬНО НАГРЕТОГО ГАЗА В ВЕРТИКАЛЬНОМ КОЛЬЦЕОБРАЗНОМ ЗАЗОРЕ ВЫНУЖДЕННО-ЕСТЕСТВЕННОЙ КОНВЕКЦИЕЙ

**Аннотация**—Анализируется численным методом теплоотдача к ламинарному потоку сильно нагретого газа, идущего вверх или вниз в вертикальном кольцеобразном зазоре, совместной вынужденно-естественной конвекцией при одновременном развитии гидродинамического и теплового пограничных слоев. При этом внутренняя стенка считается изотермической или стенкой с тепловым потоком, одинаковым на всех точках поверхности, а наружная стенка—адиабатической. Физические параметры среды считаются функциями температуры. Исходя из приближенной оценки пограничных слоев и решая уравнения неразрывности, энергии и движения, а также интегрированное уравнение неразрывности, получена характеристика теплоотдачи. В частности рассмотрено влияние изменения физиических параметров на условия возникновения обратного потока, величины критерия Нуссельта и коэффициента трения.